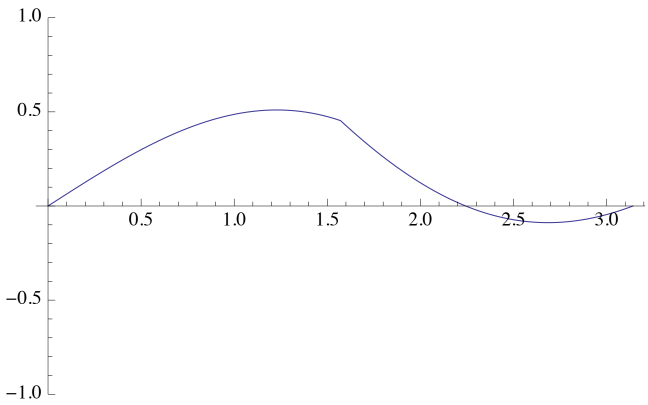
```
In[3]:= NN = 100; (** number of  intervalle for [0,π], so we get NN+1 points! **)
        FF = 30; (** number of fourier coefficients **)
```

```
In[5]:= xPos[i_] := i π/NN;
        HH := xPos[1] - xPos[0];
```

Let's choose some arbitrary initial condition for testing ...

```
In[7]:= u0 = Table[Cos[ 2.2 i / NN] 2/NN Piecewise[{{i, i < NN / 2}, {NN - i, True}}], {i, 0, NN }];
        ListPlot[{xPos[#], u0[[# + 1]]} & /@ Range[0, NN], Joined → True, PlotRange → {-1, 1}]
```

Out[8]=



The fourier coefficients are (i guess)

$b_n = \int_0^{\pi} g(x)\sin(\text{nx})\,dx$, because we have a discrete function this simplifies to

$b_n = \sum_{i=1}^{N} \int_{x_{i-1}}^{x_i} \left(g_{i-1} + (g_i - g_{i-1}) \frac{x - x_{i-1}}{x_i - x_{i-1}} \right) \sin(n\,x)\,dx$

The inner integral can be solved explicitly

(where $x_a = x_{i-1}$, $x_b = x_i$ and $g_{i-1} = g(x_a)$, $g_i = g(x_b)$, $h = x_b - x_a$):

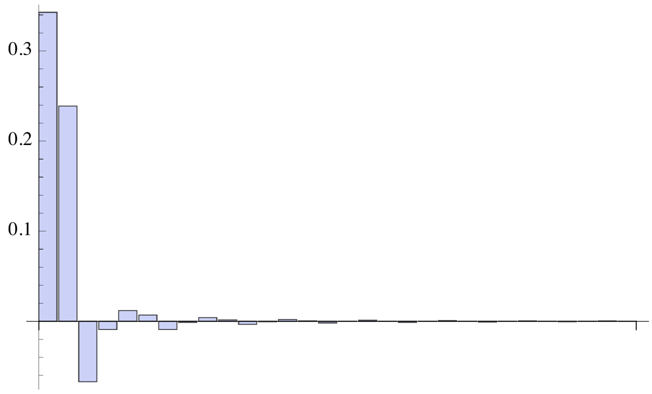
```
FullSimplify[∫_xa^xb (ga + (gb - ga) * (x-xa)/(xb-xa)) Sin[n x] dx,
 Reals,
 TransformationFunctions → Prepend[
   Function[x, x - #] & /@ Flatten@Map[{#, -#} &, {xb - xa == h} /. Equal → Subtract], Automatic]
]
```

$$\frac{\text{ga h n Cos[n xa]} - \text{gb h n Cos[n xb]} + (\text{ga}-\text{gb})\ (\text{Sin[n xa]} - \text{Sin[n xb]})}{\text{h n}^2}$$

We can (if we wish to do so) use this term to directly compute the fourier coefficients:

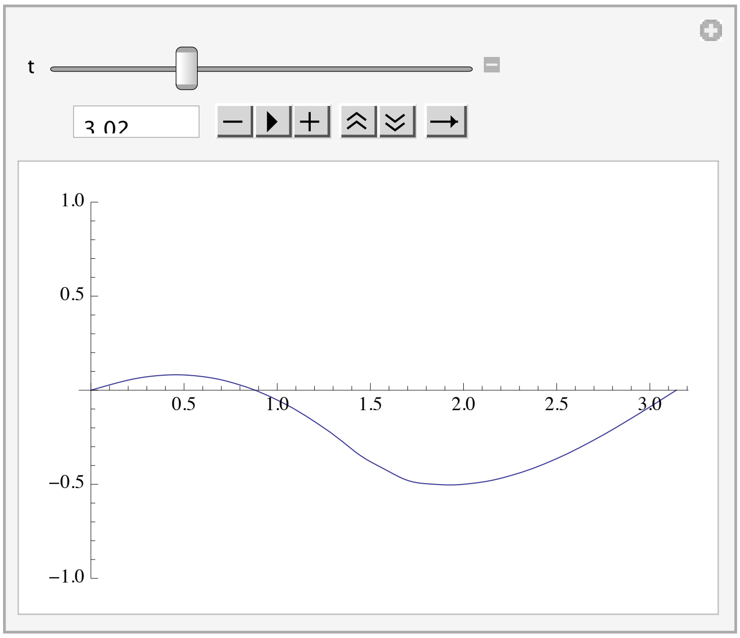
In[9]:=
```
b = Table[
   N[2/π 1/(n^2 HH) Σ_{i=1}^{NN} (u0[[i]] HH n Cos[n xPos[i - 1]] - u0[[i + 1]] HH n Cos[n xPos[i]] +
        (u0[[i]] - u0[[i + 1]]) (Sin[n xPos[i - 1]] - Sin[n xPos[i]]))],
   {n, 1, FF}
   ];
b // BarChart
```

Out[10]=



The solution is then:

```
Manipulate[
 Plot[Σ_{n=1}^{FF} b[[n]] Sin[n x] Cos[n t], {x, 0, π}, PlotRange → {-1, 1}],
 {t, 0, 10}
]
```



We can look at the solution for all times directly:

```
Plot3D[
 Sum[b[[n]] Sin[n x] Cos[n t], {n, 1, FF}], {x, 0, π}, {t, 0, 10},
 AxesLabel → {x, t, u},
 Mesh → {0, 11}
]
```